

PART B:

$$F(x)=e^{-t} \text{ } -1<t<1$$

$$F_n(x)=a_0+\sum_{k=1}^{k=n}\Big(a_k\cos(kx)+b_k\sin(kx)\Big).$$

$$a_0=\frac{1}{2L}\int\limits_{-L}^LF(x)\cos(n*\frac{pi}{L}*t)dx$$

$$a_n=\frac{1}{L}\int\limits_{-L}^LF(x)\cos(n*\frac{pi}{L}*t)dx$$

$$b_n=\frac{1}{L}\int\limits_{-L}^LF(x)\sin(n*\frac{pi}{L}*t)dx$$

$$a_0=\frac{1}{4}\int e^{-t}dt$$

$$=-e^{-t}+C$$

$$\frac{e-e^{-1}}{4}$$

$$a_0=0.5876005968219007$$

$$a_n=\frac{(e^2+1)\pi n\sin(\frac{\pi n}{2})+(2e^2-2)\cos(\frac{\pi n}{2})}{e\pi^2n^2+4e}$$

$$b_n=-\frac{(2e^2+2)\sin(\frac{\pi n}{2})+(1-e^2)\pi n\cos(\frac{\pi n}{2})}{e\pi^2n^2+4e}$$